Is ‘No’ a Force-Indicator? Yes, Sooner or Later!

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Abstract. This paper discusses the philosophical and logical motivations for rejectivism, primarily by considering a dialogical approach to logic, which is formalized in a Question-Answer Semantics (**QAS**). We develop a generalised account of rejectivism through close consideration of Mark Textor’s arguments against rejectivism that the negative expression ‘no’is never used as an act of rejection and is equivalent with a negative sentence. In doing so, we also shed light upon well-known issues regarding the supposed non-embeddability and non-iterability of force indicators. We finish by highlighting the benefits of our approach with regard to the categoricity of logics, and also a conditional that intends to solve the Frege-Geach Problem.

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1. Introduction

This paper offers a generalised account of rejectivism. To do so, we first provide an analysis of conditions it is thought that any such account must meet, by considering Mark Textor’s [32] opposition to rejectivism, summarised as follows:

> I will argue that “yes” and “no” embed in answers to propositional questions. Hence, they are not force-indicators and one cannot abstract a sign of rejection from negative answers to propositional questions.

Our reply will consist of three steps. First, we argue that affirmation and negation are prostatements rather than “yes” and “no” being prosentences

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The order of author’s names is arbitrary.

(as Textor claimed). Second, we challenge the view according to which non-embeddability and non-iteratibility are necessary preconditions for being a force indicator. Third, we propose a better formalism for setting out the dialogical foundation of logic, which includes a strengthened definition of the conditional that is capable of dealing with Textor's arguments against the standard conditional. We finish by highlighting the advantages of our approach in relation to other forms of rejectivism, with particular attention to the response to Textor given in [11].

1.1. Rejectivism

Typical attempts to construe an inferentialist theory of meaning are unilateralist, providing an account of meaning in terms of the validity of certain inferences, which is itself explained in terms of the conditions under which certain propositions may be asserted [4, 18]. Bilateralism is the view that meaning is, rather, to be construed in terms of the conditions on both assertion and denial. So, bilateralism is a form of rejectivism, which takes rejection to be on a par with, and equally foundational as, assertion [19, 22, 30]. In particular, this view takes it that the grasp that we have on speech acts of assertion and denial is prior to (in the order of explanation) our understanding of negation. So, rather than derive the rejection of p from the assertion of $\neg p$, in fact $\neg p$ is to be explained in terms of the fundamental role of rejection.[1]

In order to adequately account for these dual roles, the rejectivist must distinguish between force and content. For any content, it should be possible to mark it with the force of assertion, or the force of denial. In Rumfitt [25], this is formalised with "signed" sentences $+p$ and $-p$, signifying the force-markers on sentential metavariables. According to Rumfitt, this should be taken to formally capture the English language form of a question-forming sentence p?, with the answer "yes" or "no".[2] As such, rejectivism is not merely supposed to be a formal account, but also to capture something of ordinary linguistic practice.

By way of explanation, our own view builds upon this view with the slogan that: "Meaning is not only, but eventually Use".[3] Our view is that rules make sense insofar as they are understood as a normative theory on the manipulation of speech-acts within a question-answer game. In brief, sentences are the materials used to play this game, and rules are introduced in order for the game to make sense, but sentences make no sense when there is no speaker to do something with them. So, the implementation of inference

[1] An immediate benefit is that it becomes possible to construct an inferentialist approach to classical logic, formalised by a signed calculus [30], or a multiple-conclusion calculus [22]. In contrast, here we begin from a dialogical approach to logic based on rejectivism, and consider the various logical structures that may be constructed in this context (further work in this area can be found in [26, 33]).

[2] The formalism adopted here does not follow Rumfitt's "signed" sentences, but rather provides a direct framework for approaching answers to question-forming sentences.

[3] "Eventually" echoes with: "sooner or later" in the title: the status of proto-statements or embedded 'yes' - 'no' answers inside prostatements, before these become prostatements themselves.

rules through the expression of sentences (or sentential contents) provides an essential explanation of the role of logic with respect to the compatibility and incompatibility of speech-acts.

By way of analogy, we may consider an outfield football player to be required to play in two possible ways, namely: attacking, or defending. What else? These two attitudes are not to be compared with truth and falsity but, rather, with assertion and denial as two different attitudes towards the same goal: scoring once more than the opponent, in football; taking correct decisions, in logic.[4] It is taken for granted that a player cannot attack and defend in one and the same action at once. Yet, a midfield player may be considered as an intermediary position whose function varies with the context: as a support for the defenders, when his team does not have the ball; as a support for the strikers, when it does. Against a black-and-white view of roles in a team, we similarly want to consider a more graded view of speech-acts with assertion and denial. Then, the implementation of inference rules through the expression of sentences (or sentential contents) gives an essential explanation of this and lies at the core of logic as we understand it.[5]

In this context, rejectivism may be understood to provide coherence conditions over the game as a whole.[6] Nonetheless, it is clear that for rejectivism to even get off the ground, it must be the case that “no” can operate as a force-indicator, in at least some cases. According to Textor, however, if “no” is a force-indicator, then it fails a version of the *Frege-Geach* problem. If “no” is not a force-indicator, then it is simply a prosentence that may be construed as an assertion of a negation. Either way, rejectivism fails.

1.2. Question-Answer Semantics (QAS)

To clarify these issues, rather than work in the signed sentences framework, we work with the original suggestion that “yes” and “no” can be understood in a question-answer framework. Textor [32] calls for a better regimentation and generalised structure by which to account for rejectivism. Here, we provide such a full and generalised account by means of **QAS**. Let us briefly introduce the general framework here.

Following the inferentialist context of this discussion, we distinguish between sense and reference [4]. However, our framework is explicitly dialogical in its approach to a logic based primarily on speech acts. This is formalized by means of a generalised rejectivist-minded semantics based on yes- and

[4] For elanboration of these points, see [autor reference omitted].

[5] What follows therefore builds upon the dialogical tradition in logic, an overview of which is in [12], whilst differing in terms of the formal structure so that we have more flexibility than standard particle rules for negation.

[6] For example, Ripley [24] takes the following conditions as fundamental:

- The assertion of α and the denial of α are incompatible speech-acts;
- Having a settled opinion about α requires that one is willing to either assert α or deny α.

Whether or not these are ought to be assumed, we consider in §2.1.

no-answers to corresponding questions (**QAS**).[7] In brief, the sense of a sentence is a set of questions related to its content, and the reference is a set of correlated answers. So, on this view, $\mathbf{Q}(p)$ expresses the sense of an arbitrary sentence p and $\mathbf{A}(p)$ denotes its reference. In relation to Frege's philosophy of language, our view departs in that a reference of a sentence is not a traditional truth-value i.e. an objective and single property of a sentence. Rather, it is a relatively complex answer made by a speaker in terms of yes-no answers, where the complexity of an answer depends upon the speaker's criteria of justification for a sentence.[8] The reference of a sentence is a logical value, but it is not strictly speaking a truth-value: a sentence may be uttered as the content of an order, request, or even fear to be expressed in a that-clause.[9] Rather, logical values are a combination of yes-no answers to questions concerning an utterance, and each single answer is either affirmative or negative. Accepting and rejecting a given sentence are thus expressed by an affirmation ("yes") and a denial ("no"), but these answers do not exclusively concern declarative acts; they can be applied to any other speech act. The concept of truth is not absent from this assertoric logic, but it is no longer embedded into the reference of sentences; rather, it is included in sentences' sense, since truth occurs as the predicate of the corresponding question. More importantly, not only one question is put by $\mathbf{Q}(p)$: if the speaker gives their opinion about the truth-value of a sentence p, they need not assert either its truth or its falsehood. They may still make a conjecture, or doubt it. In the rest of the paper, our attention will be focused on a single pair of speech-acts, namely: assertion, and rejection. More especially, we will make use of $\mathbf{AR}_4$ (**A** for acceptance, and **R** for rejection) as a four-valued logical system that make senses of yes- and no-answers in such dialogical situations.

This formal device has a threefold purpose. First, it focuses on the two-sided aspect of meaning through the two independent speech-acts of assertion and denial. Second, it intends to show this striking feature (as a Janus face) within an ordered pair of valuations $\mathbf{A}(p) = \langle \mathbf{a}_1(p), \mathbf{a}_2(p) \rangle$, each of which combine various attitudes towards opposite sentences such as p and $\neg p$. Third, it brings to light an asymmetry *within* truth and falsity, on the one hand, and assertion (truth-claim, when positive; falsity-claim, when negative) and rejection on the other. Our point is that the latter attitude is not a single one and can be split into a strong reading (as a commitment of negative assertion, or falsity-claim) and a weak reading (as a mere denial

[7] We take it that the motivations for **QAS** accords both with Smiley's [30] and Textor's [32, p.447] philosophical construal of rejection. See [26] for further details.
[8] Nonetheless, the distinction between sense and reference explains how judgments can be embedded, where, in section 2.3, it will be argued that such embedding reproduce the Fregean account of opaque contexts or referential opacity, according to which a reference turns into a sense once it is in a specific context of discourse. That is: any judgment embedded into another judgment turns into a sentence, whilst the entire judgment still makes sense.
[9] For a corresponding account of truth and falsity, see [2].

of value-claim).[10] So, with respect to the degree of force, affirmation and denial may be either strong or weak, depending upon the speaker's answers. Moreover, with respect to sentential content, affirmation and denial may be either affirmative or negative, depending upon the sentential contents. As such, we can give the following clauses:

> Affirmation is a speech act such that $\mathbf{a_i}(p) = 1$
> Assertion is a strong affirmation, i.e a speech act such that $\mathbf{a_1}(p) = 1$ and $\mathbf{a_2}(p) = 0$
> Rejection is a speech act such that $\mathbf{a_i}(p) = 0$
> Negative assertion is a strong rejection, i.e., a speech act such that $\mathbf{a_1}(p) = 0$ and $\mathbf{a_2}(p) = 1$

Rejectivists stress the logical independence of assertion and rejection, such that rejection is not reducible to the act of assertion applied to a negative sentence. Correspondingly, our question-answer game requires at least two main questions to characterize the meaning of a sentence (that is, in its inferential use). These are asked to a speaker, where the expression of "being" true is relative to the speaker's criteria of truth-ascription as follows:

"Is p true?", i.e. "Is the truth of p asserted?": $\mathbf{q}_1(p)$
"Is p false?", i.e. "Is the falsity of p asserted?": $\mathbf{q}_2(p) = \mathbf{q}_1(\neg p)$

At least two sorts of answers are available to the speaker for each question $\mathbf{q}_i(p)$, namely:

"Yes", i.e. assertion: $\mathbf{a}_i(p) = 1$
"No", i.e. rejection: $\mathbf{a}_i(p) = 0$

Moreover, we also have the following equivalence rule for questions, to the effect that an answer to the truth (falsity) of an affirmative sentence p is equivalent to an answer to the falsity (truth) of its negation $\neg p$. For instance, asking "Is p false?" is equivalent to asking "Is $\neg p$ true?". In symbols: $\mathbf{q}_2(p) = \mathbf{q}_1(\neg p)$; likewise, answering "$p$ is false" is equivalent to answering "$\neg p$ is true". In symbols: $\mathbf{a}_2(p) = \mathbf{a}_1(\neg p) = 1$.

2. Entrenchment, Embeddability, and Iterability

We begin by considering, and challenging, the three major arguments against rejectivism given in [32]. These are not supposed to be exhaustive of the possible arguments against rejectivism, but they serve to sharpen and motivate the desiderata for our generalised version of rejectivism.

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[10] This bears some resemblance to Restall's construction [23] in terms of bi-theories. But note that we do not write-in restrictions on combinations of valuations by means of structural rules.

2.1. Entrenchment (i)

Textor [32, p.448] proposes a characterization of declarative speech acts as follows:

> The speech-act of rejection or denial (dissent) is taken to be the opposite of assertion; rejecting that p is distinct from and more basic than asserting that it is not the case that p.

Let us make use of our formalism to clarify this definition. By an assertion (rejection), it is meant a yes-(no-)answer to a question concerning a given sentence p. Then for any ith question about p, we have respectively $\mathbf{a}_i(p) = 1$ for assertion and $\mathbf{a}_i(p) = 0$ for rejection. Their "opposition" lies in the single values 1 and 0, accordingly, while nothing is yet said about the opposition between the sentences p and $\neg p$. The core point of rejectivism outlined by Textor is that $\mathbf{a}_i(p) = 0$ is more basic than $\mathbf{a}_i(\neg p) = 1$, because $\neg p$ proceeds from p by its negative assertion. With this depiction we agree wholeheartedly. However, Textor goes on to add the following bivalentist assumptions to his characterisation of rejectivism:

> a rejectivist account of the meaning of "It is not the case that" would be given along the following lines: "It is not the case that p" is correctly assertible iff "p" is correctly deniable. "It is not the case that p" is correctly deniable iff "p" is correctly assertible.

In a nutshell, Textor claims that every assertion of a given sentence $\neg p$ is tantamount to a denial of its opposite affirmative sentence p:

1. $\mathbf{a}_i(\neg p) = 1 \Leftrightarrow \mathbf{a}_i(p) = 0$

But, this assumption is not neutral, since a paraconsistent agent takes into account evidence both for and against p without settling between them, for want of a definite reason do to that. For them, $\mathbf{a}_i(p) = \mathbf{a}_i(\neg p) = 1$ makes sense and results in a non-bivalent value $\mathbf{A}(p) = \langle 1, 1\rangle$, where the first positive answer $\mathbf{a}_1(p)$ is about whether p is true and the second, $\mathbf{a}_2(p)$, is about whether $\neg p$ is true.[11] So, (1) holds only in a bivalent domain of values $\{10, 01\}$, where truth- and falsity-claims are exclusive to each other. And, whilst rejectivism was developed primarily in the context of classical logic, we see little reason to assume such a bivalent constraint.[12] In light of this, we may question what Textor means by "correctly" in the above definition. For example, if "correctly" invokes some sort of normality condition for truth-ascription, then it is arguable that this is inequivalent for different agents (whether bivalentists or not-bivalentists, including paracompletists and paraconsistentists). Perhaps, instead, it invokes a "rationally" condition, though this is notoriously difficult to pin-down.[13] Even so, we see at least three different criteria for the

[11] For the sake of simplicity, we will typically simplify the ordered pair $\langle x, y\rangle$ by the notation xy.

[12] Though, see [16, 24] for argument in favour of this "negation-equivalence thesis".

[13] Though, see [22] for an account of bilateralism in terms of rational "coherence" conditions.

correctness for assertion (truth-claims) and rejection (falsity-claims), following those given in [26]. Thus, the so-called Bivalentists, Paracompletists and Paraconsistentists subscribe to distinct conditions of truth-ascription with variable grades of involvement: strong (truth as a conclusive evidence), mild (truth as a state of affairs), and weak (truth as an evidence).[14] Textor neglects the occurrence of such various constraints in the theory of meaning, whilst we will develop these in our generalisation of rejectivism. Let us consider this in more detail. For example, the second clause of the above quotation may be formalized as:

2. $\mathbf{a}_i(\neg p) = 0 \Leftrightarrow \mathbf{a}_i(p) = 1$

where the values of (1) have been merely permuted. However, a Paracompletist agent may deny p without thereby asserting $\neg p$, for want of any conclusive evidence of genuine proof. For them, it is possible that $\mathbf{a}_i(p) = \mathbf{a}_i(\neg p) = 0$, resulting in a non-bivalent value $\mathbf{A}(p) = \{0, 0\}$. So, (2) holds only in a bivalent domain of values $\{10, 01\}$. Whilst this may hold in some accounts of rejectivism or bilateralism, we do not accept this constraint here, wherein denying a sentence is equivalent with asserting its negation. For example, take a question:

$$\mathbf{q}_i(p) \Rightarrow (\mathbf{a}_1(p) = 1 \text{ or } \mathbf{a}_1(p) = 0).$$

Following our rejectivist stance, there are two independent answers at hand to each such question, and what makes an answer "correct" depends upon the conditions of truth-ascription assumed by the speaker. Also, there are at least three main sorts of agents characterizing three different criteria of "correctness" for their statements:

Paracompletists (truth as having a absolute, exclusive evidence)
The agent:
cannot assert both an affirmative sentence p and its negation $\neg p$:
$\forall p \mathbf{a}_1(p) = 1 \Rightarrow \mathbf{a}_2(p) = 0$;[15]
can deny both an affirmative sentence p and its negation $\neg p$:
$\exists p \mathbf{a}_1(p) = \mathbf{a}_2(p) = 0$.
Bivalentists (truth as being the case)
The agent:
cannot assert both an affirmative sentence p and its negation $\neg p$:
$\forall p \mathbf{a}_1(p) = 1 \Rightarrow \mathbf{a}_2(p) = 0$;
cannot deny both an affirmative sentence p and its negation $\neg p$:
$\forall p \mathbf{a}_1(p) = 0 \Rightarrow \mathbf{a}_2(p) = 1$.
Paraconsistentist (truth as having a relative, inclusive evidence)
The agent:
can assert both an affirmative sentence p and its negation $\neg p$:

[14] Relevant discussion of rationality conditions in the context of paraconsistent logic can be found in [20]. As we show in §4, these difference between agents consists in enlarging or restricting the set of possible values in our semantics.

[15] So as to keep notation uncluttered, here, and throughout, we use $\Rightarrow$ as a metalanguage symbol expressing the entailment relation to indicate *therefore*, with interpretation left to context.

$\exists p \mathbf{a}_1(p) = \mathbf{a}_2(p) = 1$;
cannot deny both an affirmative sentence p and its negation $\neg p$:
$\forall p \mathbf{a}_1(p) = 0 \Rightarrow \mathbf{a}_2(p) = 1$.

However stringent or tolerant these criteria of correctness may be, one common rule is assumed in **QAS** among all the agents to rule the set of actions with speech acts. We call this *coherence*, according to which no speaker can perform both a given speech-act and its opposite about the same sentence:

$\forall p \mathbf{a}_i(p) = 1 \Leftrightarrow \mathbf{a}_i(p) \neq 0$.[16]

It is clear, therefore, that our version of rejectivism is far less restrictive than that assumed in [32]. For example, our formalism can account for the following reconstruction of Textor's defining conditions:

(i)+(ii). "It is not the case that p" is correctly assertible iff "p" is correctly deniable: $\mathbf{a}_2(p) = 1 \Leftrightarrow \mathbf{a}_1(p) = 0$; that is:

(i). $\mathbf{a}_2(p) = 1 \Rightarrow \mathbf{a}_1(p) = 0$, and:

(ii). $\mathbf{a}_1(p) = 0 \Rightarrow \mathbf{a}_2(p) = 1$.

(i)+(ii). hold for a bivalentist, but not for a paracompletist and a paraconsistentist

(i). holds for a bivalentist and a paracompletist, but not a paraconsistentist

(ii). holds for a bivalentist and a paraconsistentist, but not a paracompletist.

(iii)+(iv). "It is not the case that p" is correctly deniable iff "p" is correctly assertible: $\mathbf{a}_2(p) = 0 \Leftrightarrow \mathbf{a}_1(p) = 1$; that is:

(iii). $\mathbf{a}_2(p) = 0 \Rightarrow \mathbf{a}_1(p) = 1$, and:

(iv). $\mathbf{a}_1(p) = 1 \Rightarrow \mathbf{a}_2(p) = 0$.

The same conclusions hold for (iii) and (iv) as for the first quotation (i)-(ii).

So, these various conditions lead to a number of different statements for the speakers. First, the Paracompletist would say "It is not the case that p" is correctly assertible *only if* "p" is correctly deniable. Second, the Bivalentist would say "It is not the case that p" is correctly assertible *if and only if* "p" is correctly deniable. And thirdly, the Paraconsistentist would say "It is not the case that p" is correctly assertible *if* "p" is correctly deniable. These differences cannot be ignored from a rejectivist perspective, so to assume Bivalentism for this purpose is not an acceptable argument against Rejectivism in general.

Due to this, although we agree with Textor about the general framework within which assertion and rejection make sense, we disagree about the way these are to be used.

2.2. Entrenchment (ii)

Textor also provides an argument against rejectivism by appeal to the use of "no" in vernacular English. For example, Textor [32] argues that:

> The sign of rejection '. . . ? No' is introduced as an abstraction from a negative answer to a propositional question.

[16] See also Section 2.1, formula (3) Cf. Rumfitt or Ripley, who mentioned something similar whilst the latter admits "paracoherence"; see also [33] for a detailed discussion of coherence and paracoherence in the context of constructive logics.

We agree, since this accounts for Frege’s famous distinction between “judgeable contents” (sentences) and “judgments” (statements). For, what Textor means by a “propositional question” corresponds to what Frege depicted as an invitation to acknowledge the truth of a thought through the expression of a sentence, without the latter being already claimed to be true or false. In other words, a Fregean sentence p is a propositional question “p?” and the sense of a statement in **QAS**: $\mathbf{q}(p)$.

However, we depart from Textor regarding the meaning of “no”: “no” crucially differs from “not”, just as an answer clearly differs from a question. Indeed, we take the former to be, for example in the case of $\neg p$, an abstraction from a propositional question about whether p is rejected (taken to be false). Then the symbol ‘$\neg p$’ corresponds to ‘$\mathbf{q}(\neg p)$’ in our formal language, and it is still not an answer as it stands. Rather, there are two ways of rejecting p: either by asserting its falsity or by rejecting its truth, keeping in mind that both answers are different (*pace* Textor’s previous bivalent characterization of rejectivism). This stands in opposition to Textor’s assumption that only one sense of “no” prevails in ordinary language:

> Our understanding of this sign and the plausibility of the claim that it is a sign of rejection depends on our prior understanding of the English word ‘no’ in its use in answering propositional questions (...).

To this, we reply in two ways. On the one hand, the prominent role given by Textor to ordinary language in order to make his point against rejectivism is at odds with another broader purpose of formalization, namely: regimentation. This has been stressed by Incurvati & Smith [11, p.226], concerning the legitimate arbitrariness of denial as a abstract sign in logical theories. Making use of the vernacular allows Textor to apply a kind of *reductio* argument against any minor uses of “no”. But, consider the case of a mere withdrawl, as symbolized by the third “gappy” value in many-valued systems (typically symbolised by n, $1/2$ or i). Should the anti-rejectivists discard the existence of such non-classical values to make their point? It does not seem so, since these non-bivalent values are so firmly entrenched in the history of logic (from Aristotle’s sea-battle onwards) that we hardly see how denial could be ignored in its natural and intuitive occurrence as a mere absence of judgment or non-commitment.[17]

As such, Textor’s criterion of linguistic entrenchment cannot but favor Bivalentism and anti-rejectivism, where yes and no-answers are not taken to be independent from each other. We take this logical dependence to be traced back to Grice’s Maxim of Quantity: that, as a matter of fact, we do

[17]This garners support from the well-known pragmatic ambiguity of ordinary language negation. See [31] for cases in which the use of “not” in a natural language context indicates the rejection of an assertion without also indicating the assertion of the negation of the relevant sentence; e.g. “Some men are not chauvinists. All of them are”, “John isn’t wily or crazy. He’s wily and crazy”.

not answer "no" without thereby committing to the truth-value of the questioned sentence. This echoes Frege's equivalence Thesis, exemplified by the preceding formulas (1) and (2). But, a common example may show how our natural phrasing leads one astray with bivalent-minded words. For example, the vacillating expression "yes and no" does not mean what it overtly says: it does not mean both a yes- and no-answer to one and the same answer:

3. $\mathbf{a}_i(p) = \{1, 0\}$

but, rather, two yes-answers to the separate questions about p's truth and its opposite $\neg p$:

4. $\mathbf{a}_1(p) = \mathbf{a}_2(\neg p) = 1$

as the case is when a speaker is undecided about p, so sustaining the weak statement $\mathbf{A}(p) = 1, 1$. In other words, the words "no" in "yes and no" may allude to a "yes"-answer related to the opposite sentence. It "can" or "ought to" do so, if we take (3) to be a genuine principle of incorrectness (as we will).

Consider, further, the role of vernacular expressions that Textor uses to support his anti-rejectivism. For example, Textor refers to the *OED* in order to explain the meaning of yes-answer:

> The *OED* says: Yes: In answer to a question not involving a negative; standing for the affirmative sentence corresponding to the interrogative one constituting the question: 'It is so'.

Actually, the aforementioned definition may hold in English but not in other natural languages. In which case, not every ordinary yes-no answer complies with the same rules in different natural languages. For example, Russian speakers can give a yes-answer ("da") to confirm a propositional question that involves a negative statement.[18] To the question "You haven't done your homework, have you?", the response may be "yes", to validate the content of that propositional question involving a negation; in the contrary case, the response may be "no", to invalidate the reproach implied by the questioner.[19] So, a Russian speaker answers positively to confirm the speaker's opinion that $\neg p$ is true, where English speakers would normally say "no".[20]

Here is a fruitful feature of entrenched (but not regimented) yes-no answers: Russian answers are about speakers' statements, rather than the sentential content stated. In a nutshell, Russian speakers say "yes" or "no" in order to express bare agreement or disagreement, respectively and regardless of the logical form of the sentence. This also suggests that the question is not plainly univocal. Indeed, there seems to be a relevant difference between

[18]By "yes" here, we mean the type particle indicating affirmation, i.e. the positive polarity particle in the Russian language. We thank an anonymous reviewer for prompting this clarification.

[19]In Russian, this is either "vi nie sdielali" (polite form), or "ti nie sdielal" (colloquial form). In full; "Ti nie sdielali svayou damachnyouyou rabotou, nie tak li?"

[20]We use the Russian sample here only as a way to question Textor's view of the no-answer. However, it is worth clarifying that we want to deal with yes- and no-answers as a form of universal linguistic mechanism.

two sorts of questions about a given sentence $\neg p$, namely: question as an information-seeking device, where the questioner doubts whether $\neg p$ is the case; and question as a confirmation-seeking device, where the questioner believes that $\neg p$ is the case (or, equivalently, p is not the case). Russian speakers would answer "*da*" to confirm (the truth of) $\neg p$ in the second case, whence the meaning of "*da*" and "*net*" correspond to confirmation and refutation of sentential contents. Accordingly, we think that there is little symmetry between natural languages in this respect. For example, in the example above, a French affirmative "*si*" rejects $\neg p$, and corresponds to the Russian "*net*"; in symbols: $\mathbf{a}_1(\neg p) = \mathbf{a}_2(p) = 0$. Whereas, the French "*non*" confirms $\neg p$ and corresponds to the Russian "*da*"; in symbols: $\mathbf{a}_1(\neg p) = \mathbf{a}_2(p) = 1$.

So, at the least, Textor's argument from the vernacular is on less than stable ground. On consideration of Textor's entrenchment argument, our first conclusion is that vernacular language is neither a necessary nor a sufficient condition to conclude that "no" equates with negative assertion. Let us now turn to Textor's argument from embeddability.

2.3. Embeddability

According to Textor [32, p.448], no logical term can operate as a force-indicator once embedded into a sentence:

Non-embeddability. Force-indicators cannot be embedded. If '$\neg$' is a sign of rejection in a language L, sentences of the form '$\neg p$' never occur as semantically significant parts of complex sentences of L.

We take non-embeddability to be an unjustified assumption. However, this is dialectically tricky, since it is also accepted in the rejectivist literature, so, we must explore this further. Rejectivism has typically relied upon a distinction between force and content, and, since embedding force "makes" force into a content (by the definition of embedding), this distinction would fail should force be embeddable. There is, nonetheless, a possible route around this argument by paying closer attention to the roles of force in the context of questions and answers. For example, according to Textor;

> The article "yes" is a prosentence, and if "yes" is a prosentence, "no" should also be one. Answering "no" is another way of asserting "It is not so".

Rather, we reverse these terms: affirmation is a pro*statement*; and if affirmation is a pro*statement*, then negation should also a pro*statement* within sentences. On this view, uttering "not" (in a question) is a shorter way of asking "Is it not the case that ... ?".

First, let us explain two neologisms with the required caution any such process should demand: *prostatement*, and *proto-statement*. Unlike Bolzano's prosentence, by a "prostatement", we mean the status of the expressions "yes" and "no" when occurring as a placeholder for a whole statement. By a "proto-statement", we mean the status of these expressions when embedded into a statement such as a conditional or disjunction.[21] These are not yet speech-acts, but they come to be so sooner or later, if the whole statement makes sense by its use. That is to say, sentences are nothing but proto-statements, i.e. informative items that occur in communication to express information, whether asserted or not. A sentence proceeds as a pro-statement, in the sense that its role is to be used later on in the form of an assertion or a rejection. For what is the point of asserting e.g. "if p then q", if p and q are never asserted separately?[22] This means that the content of a statement must *become* asserted itself, following the definition of meaning as use (and given compositionality).

This coheres with a wider point of view that the difference between sentences and statements is essential to characterize logic as a set of actions upon sentences. By means of our distinction between a proto-statement and a prostatement, the issue of embeddability may be overcome by introducing a more dynamic understanding of the relationship between sentences and statements within complex sentences such as conditionals. Thus, force-indicators can be embedded as proto-statements rather than prostatements: the former are not still asserted or rejected by the speaker, but they come to be so sooner or later, thereby turning into the latter expressions.

The symbolism of **QAS** makes a distinction between sentences and statements in terms of questions and answers, respectively, and in accordance with Frege's claim by analogy with the pattern of scientific inquiry.[23] So, letting p stand for "John is the murderer", we have the corresponding data:

"Is John the murderer?"
i.e. the proposition that John is a murderer: $\mathbf{q}_1(p)$[24]
"Is John not the murderer?"
i.e. the proposition that John is not the murderer: $\mathbf{q}_1(\neg p) = \mathbf{q}_2(p)$
"John is the murderer": $\mathbf{a}_1(p) = 1$
i.e. "Yes, it is the case that John is the murderer"
"John is not the murderer"

[21] Dummett's [3] distinction between *ingredient sense* and *assertoric content* may be understood as a precursor to the distinction that we make here. For further discussion in support of the distinction, see [34].

[22] We propose to extend the current account to deal with subjunctive conditionals in further work. However, we note here that, as an anonymous reviewer points out, subjunctive conditionals seem problematic for our account of proto-statements since one can have meaningful discourse involving counterfactual sentences that are never asserted as true. Our response is two-fold. First, even if counterfactuals were to resist our treatment of the Frege-Geach problem, this would not undermine the account of ordinary conditionals. Second, we think that counterfactuals may be treated similarly to suppositional conditionals, in which to test the meaning of a such conditionals, we would "suppose" the antecendent to hold, to test whether the conditional itself holds, dependent upon the antecendent holding in a suppositional context.

[23] For example, in "Negation" [9], Frege states that: 'A propositional question (*Satzfrage*) contains a demand that we should either acknowledge the truth of a thought, or reject it as false' (117).

[24] This corresponds to Frege's *Gedanke*.

i.e. “Yes, it is the case that John is not the murderer”: $\mathbf{q}_1(\neg p) = \mathbf{q}_2(p) = 1$

The main difficulty is of a vernacular order, whenever a “no”-answer is given to such questions. Although Textor [32] rightly notes that the main use of a negative answer is intended to confirm the truth of a negative sentence, above we said that this is not the only available meaning in which a no-answer can be used. Roughly speaking, a no-answer as a rejection is strong when expressed by “no, it is not the case that ...”; and weak when expressed by “no, I do not assert that it is the case that ...”.[25] The more complex structure of weak rejection echoes with our plea for the iterability of force-indicators, and also makes room for a second sense of rejection as non-commitment, rather than commitment to the truth of $\neg p$.

This explanation can be clarified through a brief analysis of the conditional. It is notoriously tricky to understand how we should be capable of asserting conditionals “conditionally”, so to speak. For example, it seems intuitively incorrect to think a conditional, $\alpha \to \beta$, is the sort of thing that is simply asserted, especially when the antecedent α is known to be false. Rather, as Humberstone [10] suggests, after Ramsey [21], to assert a conditional is not to be thought of as asserting a conditional proposition, but to make a conditional assertion of the consequent: ‘If the latter condition is not satisfied (i.e., if the antecedent is false), then it is as though no assertion had been made. A parallel can be made with conditional bets, which are void in that no money changes hands unless the condition they are conditional upon obtains’ (p.938). Edgington [6] goes further still: ‘to assert a conditional is to assert that it is true on condition that it has a truth value. To believe a conditional is to believe that it is true on the supposition that it has a truth value. It has a truth value iff its antecedent is true’. The interpretation is also argued for by Quine in [36, §3]:

> An affirmation of the form If α then β is commonly felt less as an affirmation of a conditional than as a conditional affirmation of the consequent. If, after we have made such an affirmation, the antecedent turns out true, then we consider ourselves committed to the consequent, and are ready to acknowledge error if it proves false. If on the other hand the other hand the antecedent turns out to have been false, our conditional affirmation is as if it had never been made.

We think that this view of conditionals holds more widely, and requires a distinction between proto- and pro-statements.[26] As such, our explanation is analogous to a conditional bet of the form “I bet that if p, then q”, where the

[25] Recall Grice’s Maxim of Quantity, discussed above, which provides explanation for why most, but not all, “no”-answers function as strong rejections.

[26] This also clarifies our underlying inferentialist approach to logic, in the sense that we require evidence for truth. Indeed, unlike orthodox readings of Quine’s argument, for us “conditional on the truth” means the same as “if asserted by the speaker”, rather than taking truth and falsity to be objective truth-values that are properties of propositions.

condition on which the bet rests is not a truth but a potential speech act, i.e. a proto-statement. Imagine a case in which the antecedent of a conditional is true (i.e., it is the case that p, in "if p, then q"), but the speaker does not know it to be so. Should the speaker assert the consequent under the pretext that they assert that if p then q? We do not think that this is the case since the speaker still denies p. In other words, our replacement of truth by affirmation helps to avoid a case of logical omniscience, so that, if I assert that (if p, then q), I am not required to assert q from p's truth even without asserting p. Once this distinction between proto- and pro- statements is allowed, embeddability is no longer the strict criteria once thought, since we also need to take into account the *dynamics* of use.

The distinction is mirrored by the shift from the protostatement, as conditional, and the prostatement, as implication, by dint of the deduction theorem. In many logics, the deduction theorem allows us to identify a proof with a deduction from the empty set, and a theorem as the last item in that proof.[27] As is known since Herbrand, to prove that deduction theorem holds requires induction on successive uses of Modus Ponens from the axioms, so, in effect, this process requires us to shift assumptions successively over to the *r.h.s* of the turnstile, removing the hypotheses and treating them as theorems. Perhaps more than other formulations, the categorical version of the single-premise deduction theorem makes this transparent. Take 1 to indicate algebraic top, then say that for any propositional formulas α and β, if there exists a derivation of $1 \vdash \beta$ from the assumption $1 \vdash \alpha$, then there exists a derivation of $\alpha \vdash \beta$ [13, p.50]. The illusion that this brings with it lies in the ease with which this *appears* to shift from the hypothetical to the actual form, which is surely one of the morals of Carroll's tale: that there is a distinction between the action of an inference, and a relation between propositional contents. One mistake identified in Carroll's story is to think that to infer β from α and $\alpha \rightarrow \beta$ requires an agent to believe "$(\alpha \wedge (\alpha \Rightarrow \beta)) \Rightarrow \beta)$". The latter may, rather, be that which may be the content of a belief *following* the act of making the inference itself.

2.4. Iterability

An additional issue is the criterion of iterability, the failure of which is said to follow from the aforementioned failure of embeddability in Textor [32]:

Iterability. [If $-$ were a sign for rejection] "$-$" is also non-iterable in L: there are no sentences of the form "$- - p$" etc. in L.

Whilst our plea for embeddability allows that proto-statements (but not prostatements) can be embedded into a pro-statement, iterability means that componing protostatements can be iterated into a whole prostatement. This has been commonly viewed as an absurd device, even by the rejectivists who rule out any introduction of assertions or rejections within another, such as: "I assert that I assert that p", or "I assert that I deny that p".

[27] See [5] for a similar formulation and extended discussion of deduction theorem.

Here is one way of avoiding this criterion. Borrowing from modal logics or quantification theory, the operator endowed with the largest scope plays the same role as a prostatement, while the embedded (and possibly) iterated ones proceed as proto-statements. For instance, in "a knows that a does not know that p" (in symbols: $Ka\neg Kap$), we have a knowledge-statement about a case of non-knowledge. The latter also stands for a statement, once the largest operator is eliminated through the inference rule of factivity ($Kp \supset p$), viz. $Ka\neg Kap \supset \neg Kap$. It is not typically thought that such an expression of ignorance is not an epistemic act once embedded into a larger one. We apply the same formal treatment to speech acts and, by doing so, propose an answer to the Frege-Geach problem and Textor's objections. It is not possible to iterate an act since they are expressive in the context of prostatements, whilst in protostatements, there is rather the description of an act which may then be iterated in much the same way as modal or epistemic operators. That is to say, when a judgment is embedded, it is not a judgment any longer, since it becomes a sentential content. It is in this way that we can make sense of embeddability, since it is not strictly speaking a judgment that is embedded.

Let us examine these objections. By "cannot be", is it said that any form of embedding or iteration is nonsense? Is it logically, conceptually, or epistemically impossible to have such a well-formed formula in a given language? None of these are obviously the case. For example, consider an assertoric version of excluded middle, "I assert p, or I deny p", which is a sentence of a given language comprising two speech-acts:

5. $\vdash p$ or $\dashv p$

   In **QAS**, this is:

   $\mathbf{a}_1(p) = 1$ or $\mathbf{a}_1(p) = 0$

We may also point to the device of introducing metalinguistic symbols in the object-language by means of "internalization" [17, 37]. In [37], von Wright considers T and F to be constructed as unary operators of truth and falsity, respectively. In this context, there seems little difference between (5) and the formula $Tp \vee \neg Tp$ of von Wright's logic. Now, consider the statement "It is not true that it is not true that p", which is just a rephrasing of what is supposed to be nonsense according to Textor. But, it seems we can make sense of this by introducing a truth operator as $\neg T \neg Tp$. Again, embedding and iteration can be observed both in truth-logics and modal logics in general.

The present analogy between the two sorts of logical systems is purported to show that speech acts can be also embedded and iterated without "changing their meaning", as Textor puts it, and the Frege-Geach problem alludes to.[28] Nonetheless, in making this argument, Textor refers to Rumfitt's [25] argument regarding non-iterability. There, Rumfitt argues as follows:

[28] This is not to say that these unary operators are modal operators. Rather, a number of counterintuitive results can be found in the literature against a modal treatment of many-valued systems, especially concerning the distribution of modalities over conditional or disjunction. For discussion, see [7] and [1].

> Is it the case that two is not a prime number? No" makes perfectly good sense, but "Is it the case that it is the case that two is a prime number? No? No" is gibberish. The sign "–", then, does not contribute to propositional content, but indicates the force with which that content is promulgated. (pp.802-3)

The use of such double negative expressions may strike one as very counterintuitive. Though, again, there is an analogy to be made with modal systems such as epistemic or alethic logics, where such awkward but meaningful formulas as "I know that I know that I know that p" or "It is necessarily impossibly necessary that possibly p" are logically well-formed. If the latter are taken to be meaningful, despite their unusual occurrence in the natural language, we think that this should equally be the case for iterated speech acts. To a degree, Rumfitt's argument against iteration is on a par with a reduction thesis, in order to simplify and make corresponding formulas more intelligible. But, that they are only vaguely intelligible at first utterance does not make them meaningless.

Despite appearances, the sentence quoted by Rumfitt is not "gibberish" from a logical point of view, but rather appears to be meaningless only due to its unusual grammar. For, "Is it the case that it is the case that two is a prime number? No? No" can be split into a question and an answer. The question goes from the initial sentence to the question mark, and can be rephrased as a negative question including a affirmative sentence as: "Is it the case that it is the case that two is a prime number?". The answer is negative: it is a no-answer expressed by the exclamation mark. That is: "No, it is not the case that is the case that two is a prime number". The main problem with such an intricate sentence lies in the embedded form of the question: "it is not the case that it is the case". But again, a parallel with modal logics shows that reduction laws help to disentangle these heavy formulations by rendering them equivalent with simpler expressions. In epistemic logic, for instance, iterated knowledge can be reduced to mere knowledge by applying the theorems S4 or S5. To sum up: we agree with Rumfitt that we may consider only one speech act at once in the case of iterated formula; we disagree that the narrow operators do not stand for speech acts any longer, just as the second modal operator K_2 still stands for a proper operator of knowledge in the iterative formula K_1K_2p. The upshot is that any *simultaneous* iterability is not possible, in the same sense that the formulas $Ka\neg Kap$ (or $T\neg Tp$) cannot express both knowledge and non-knowledge (or truth and about non-truth) with respect to the same sentence p. But, this does not rule out iterability *tout-court*, provided that the distinction between proto- and pro-statements is clarified.

3. The Frege-Geach problem

We are now in a position to draw these arguments together in order to consider the way in which Textor [32] addresses the infamous Frege-Geach problem. First, consider Frege's test for force-indicators as utilised by Textor [32]:

> If it is claimed that E is a force-indicator, try to embed it in the antecedent of a conditional. If you can do this without changing the meaning of E, E has no assertoric force.

The construal of "meaning", here, in keeping with the original motivation for rejectivism, will presumably have the sense of inferential role or use. Then, the idea is that, whoever embeds an assertion (or rejection) inside a conditional sentence must inevitably change the inferential role of the conditional as it stands. We provide reason to think that this is not the case, by formalising Textor's examples by means of **QAS**, after first reviewing the Fregean context of the argument.

Textor begins his analysis as follows:

> For instance, does "is true" indicate assertoric force? No, it is contained in the antecedent of the conditional "If it is true that John is the murderer, Peter is innocent", but the antecedent is not asserted, only the whole conditional is.

Importantly, we suspect Textor to assume a difference between the expressions "If it is true that John is the murderer, Peter is innocent" and "It is true that if John is the murderer, Peter is innocent". We see none. That is, "it is true" is not really embedded into the entire conditional. Such a vernacular argument is captious with respect to the criterion of embedding. For instance, our formalization of Textor's sentence can be depicted in three different ways in $\mathbf{AR}_4$ (with p for "John is the murderer, and q for "Peter is innocent"):

6. If it is true that John is the murderer, Peter is innocent
   (a) $\mathbf{a}_1(p \supset q) = 1$
   (b) $\mathbf{a}_1(p) = 1 \Rightarrow \mathbf{a}_1(q) = 1$
   (c) $(\mathbf{a}_1(p) = 1) \supset q$

(6b) is nothing but the metalinguistic formulation of the clause for the assertibility-conditions of the conditional, corresponding to Modus Ponens. Not only are (6a) and (6b) equivalent with each other, from our inferential view of meaning for logical constants, but also, they nicely exemplify the embeddability of force-indicators in (6b) through the force of assertion. Is the latter really embedded in (6b) though? Isn't it rather the whole conditional $(p \supset q)$ that has force, separately from a "conditional force" inside the conditional? Once again, our reply is that a genuine force is expressed by prostatements, whilst conditional force is rendered inside proto-statements. A proto-statement comes to be a statement once the whole statement is used for the normal and essential purpose of logic, i.e. drawing conclusions by implementing the components as separate speech-acts (whether asserted or denied). At the same time, (6c) is not a well-formed formula since questions and answers are interrelated there: the first component is an answer, expressed by means of a statement and belonging to the metalanguage; the second component is a question, expressed by means of a sentence and belonging to the object-language The latter corresponds to what Textor has in mind with his "if yes"-phrase, whereas we take (6) to be correctly symbolized by both (6a) and (6b).

Returning to the connection between embeddability and iterability, our previous comparison between force-indicators and unary operators makes a link between the two rules of detachment and distributivity: just as q can be detached from p in the Modus Ponens, assertion can be distributed on the sentences p and q once conditional is asserted itself. If we refer to the conditional force of components, these become proper forces after being detached from that which plays initially the role of antecedent p, in the proto-statement "if p, then q", and then the role of a premise in the prostatement "p, therefore q". We may consider Textor's additional examples similarly to merely presuppose what is to be proved by concrete cases of negative assertion:

> 1. Is this tree deciduous? If yes, go to #2. If no, go to #4.
> 2. Are the leaves heart-shaped? If yes, this is a quaking pen. If no, go to #3.
> Importantly, in 1. and 2. "yes" and "no" have the grammatical role of complete sentences.

We disagree about the claim that "yes" and "no" are embedded here, and we propose an alternative formalization of 1. and 2. Let p stand for "this tree is deciduous" or "the leaves are heart-shaped", q for "go to #2" or "this is quaking pen", and r for "go to #4" or "go to #3". It results in the following common question-answer game in $\mathbf{AR}_4$:

7. Is it the case that p? If yes, then it is the case that q. If no, then it is the case that r.
   (a) $\mathbf{q}(p)$
   (b) $\mathbf{a}_1(p \supset q) = 1 \Leftrightarrow \mathbf{a}_1(p) = 1 \Rightarrow \mathbf{a}_1(q) = 1$
   (c) $\mathbf{a}_1(\neg p \supset r) = 1 \Leftrightarrow \mathbf{a}_1(\neg p) = 1 \Rightarrow \mathbf{a}_1(r) = 1$

The last two formulas have the same logical form, the sole difference lying in the use of a negative antecedent in (8c). There is no difference between this example and the preceding Frege's test (7a)-(7c), so our conclusion will not differ from it.

A further example given by Textor insists upon the troublesome distinction between non-committed sentences and committed statements, namely:

> (P1) Is this tree deciduous? If no, go to #4.
> (P2) No.
> Therefore:
> (C) Go to #4. (...)
> If "no" had different meaning in (P1) and (P2), we would commit a fallacy of equivocation. "No" must have the same meaning in (P1) and (P2). So if "no" is not a force-indicator in (P1), it cannot be one in (P2).

Our formalization for this case is another instance of Modus Ponens:

(P1) $\mathbf{q}(p), \mathbf{a}_1(\neg p \supset q) = 1 \Leftrightarrow (\mathbf{a}_1(\neg p) = 1 \Rightarrow \mathbf{a}_1(q) = 1)$
(P2) $\mathbf{a}_1(\neg p) = 1$
(C) $\mathbf{a}_1(q) = 1$

The equivalence relation between detached assertions in (P1) means that "no" is embedded into two different formulas with one and the same meaning in use, insofar as meaning is given as a set of norms for assertibility and deniability-conditions. If so, then "no" does proceed as a force-indicator in (P1) too, just as K_2 occurs as a box-operator in K_1K_2p, in epistemic logic. Consequently, there is no fallacy of equivocation.

4. A rejectivist-minded semantics

4.1. The logical system: $\mathbf{AR}_4$

We briefly sketch the Logic of Acceptance and Rejection ($\mathbf{AR}_4$), a 4-valued system based on the exhaustive set of four independent answers to propositional questions. Its language contains a set of sentences and a set of logical constants, namely: negation, conjunction, disjunction, and conditional, with the usual syntactic rules for well-formed formulas. Despite its striking resemblance with Belnap's First-Degree Entailment, it differs from it by including an alternative definition of conditional. Moreover, the distinction between speakers and their distinctive constraints upon correctness is rendered by a distinction in their domains of valuation. Following the preceding section, $\mathbf{AR}_4$ may be adapted to paracomplete, bivalent, and paraconsistent agents as follows:

$\mathrm{V}_4 = \{11, 10, 01, 00\}$
Paracompletism: $\mathrm{V}_{3\text{-}} = \{10, 01, 00\}$
Bivalentism: $\mathrm{V}_2 = \{10, 01\}$
Paraconsistentism: $\mathrm{V}_{3+} = \{11, 10, 01\}$

The following logical system $\mathbf{AR}_4$ is a language based on a general question-answer framework $\mathbf{AR}^n_m$: to a given ordered set of n questions corresponds an ordered set of of m kinds of answers given by an agent-speaker. The number of questions and answers relies upon the sentence in consideration, depending upon the degree of complexity of the information it conveys. In the present case, two questions are about an arbitrary sentence ϕ, namely: Is it true? Is it false? Let us notice that the model at hand is reminiscent of Belnap's First Degree Entailment, where truth and falsity do not mean the existence of facts but, rather, the occurrence of data for or against the corresponding sentence. Two kinds of answers are to be given to each of these two questions: yes, or no.

The logical system **FDE** of Belnap (1977) can be easily adapted by means of this dialogical modelization of information. Let $\mathbf{A}(\phi) = \langle \mathbf{a}_1(\phi), \mathbf{a}_2(\phi) \rangle$ be the logical value of an arbitrary sentence ϕ, where $\mathbf{A}$ is a valuation function such that **A**: $\phi \mapsto$ {1,0}×{1,0} (0 for the negative answer *no*, 1 for the positive answer *yes*).

The set of Belnap's basic four truth-values yields the following four structured values: T = $\langle 1, 0\rangle$, F = $\langle 0, 1\rangle$, B = $\langle 1, 1\rangle$, N = $\langle 0, 0\rangle$, thereby justifying the heading $\mathbf{AR}^n_m = \mathbf{AR}^2_2 = \mathbf{AR}_4$ of our many-valued logic of information.

A major difference between **FDE** and $\mathbf{AR}_4$ concerns the relation of consequence, especially the logical relation obtaining between the four truth-values. In **FDE**, there is a partial ordering relation between T, F, B, and N; this order is depicted notably in Ginsberg (1988) by a bi-lattice coposed of two distinctive ranges: the range of truth t, and the range of information i. In this two-tiered lattice, two independent ordering relations are to be distinguished from each other:

T $>_t$ B $>_t$ F
T $>_t$ N $>_t$ F,

wherein B and N are incomparable with respect to ¿$_t$ (the one is not "truer" than the other).

B $>_i$ T $>_t$ N
T $>_i$ F $>_t$ N,

wherein T and F are incomparable with respect to $>_i$ (the one is not more informative than the other).

Lattice theory plays a major role in the algebraic definition of Tarskian consequence, insofar as the relation $\phi \models_{t/i} \psi$ holds if and only if $v(\phi) <_{t/i} v(\psi)$. At the same time, there is no ordering relation in $\mathbf{AR}_4$: each logical value is independent from the three other ones, and the sole relevant notion to characterize the relation of consequence is that of designatedness: $\phi \models_{t/i} \psi$ holds if and only if $\mathbf{A}(\phi) \in D$ whenever $\mathbf{A}(\psi) \in D$, D symbolizing the set of *designated* truth-values such that $D = \{10, 11\}$.

The set of logical constants occurring in $\mathbf{AR}_4$ is characterized similarly to those of **FDE**, with the notable exception of conditionals. The set of sentences of $\mathbf{AR}_4$ are to be construed in accordance to the Backus-Naur form:

$$\phi := p \text{ — } \neg\phi \text{ — } \phi \wedge \psi \text{ — } \phi \vee \psi \text{ — } \phi \rightarrow \psi$$

These logical constants are defined with the help of the Boolean operators $\sqcap$ and $\sqcup$ applied to answer values 1 and 0, such that $1\sqcap 0 = 0\sqcap 1 = 0\sqcap 0 = 0$ and $1\sqcup 0 = 0\sqcup 1 = 1\sqcup 1 = 1$. The definitions are given as follows, where the symbols of the logical values are simplied in the general form $\langle x, y\rangle = xy$. For any sentence $\mathbf{A}(\phi) = \langle \mathbf{a}_1(\phi), \mathbf{a}_2(\phi)\rangle$:

Atoms: $A(p) = 1, 1$ or $A(p) = 1, 0$ or $A(p) = 0, 1$ or $A(p) = 0, 0$;

Negation: $A(\neg\varphi) = \langle a_2(\varphi), a_1(\varphi)\rangle$;

Conjunction: $A(\varphi \wedge \psi) = \langle a_1(\varphi) \sqcap a_1(\psi), a_2(\varphi) \sqcup a_2(\psi)\rangle$;

Disjunction: $A(\varphi \vee \psi) = \langle a_1(\varphi) \sqcup a_1(\psi), a_2(\varphi) \sqcap a_2(\psi)\rangle$;

Implication: $A(\varphi \to \psi) = \langle a_1(\varphi) \sqcap a_1(\psi), a_1(\varphi) \sqcap a_2(\psi)\rangle$;

The characteristic matrices for the above constants are similar to those of **FDE**, assuming a transcription of the symbolic letters in Belnap (1977) into structured values of $\mathbf{AR}_4$.

	$\neg$
11	11
10	01
01	10
00	00

$\wedge$	11 10 01 00
11	11 11 01 01
10	11 10 01 00
01	01 01 01 01
00	01 00 01 00

$\vee$	11 10 01 00
11	11 10 11 10
10	10 10 10 10
01	11 10 01 00
00	10 10 00 00

$\to$	11 10 01 00
11	11 10 01 00
10	10 10 00 00
01	00 00 00 00
00	00 00 00 00

Fig. 1: Matrices for $\mathbf{AR}_4$

Similarly, $\mathbf{AR}_4$ is endowed with a set of axioms and corresponding tableaux just as in **FDE**, with the notable exception of inference rules for implication: the specific behavior of the latter in $\mathbf{AR}_4$ is such that the so-called paradoxes of material implication are overcome there; however, this does not prevent its main properties to be preserved, viz. Modus Ponens (MP) and Modus Tollens (MT).[29]

The conditional is the main innovation of $\mathbf{AR}_4$, insofar as its assertibility-conditions are strengthened with respect to the mainstream definition. Here, a conditional cannot be asserted once either of its components is not also asserted. This complies with our preceding view of conditionality as a commitment upon its terms, so that the whole should be rejected once either of its components does. Above all, such an intuitive characterization of conditional can make sense only if rejectivism is admitted: without a clear-cut separation of assertion- and rejection-conditions, it clearly appears that our conditional would collapse into the other constant of conjunction, given that $\mathbf{a}_1(p \wedge q) = \mathbf{a}_1(p \to q)$. The crucial difference is made in the level of their rejection-conditions, since $\mathbf{a}_2(p \wedge q) \neq \mathbf{a}_2(p \to q)$. The last main feature of $\mathbf{AR}_4$ concerns its main relation, namely: consequence. What does it does mean exactly, from a rejectivist point of view?

4.2. Rejectivist consequence

Usually, logical consequence is defined as a relation of truth-preservation from the premises to the consequence. But the situation cannot be depicted

[29] In §4.4, we consider the difficulty of formalizing MT from a bilateralist perspective. We aim to develop a natural deduction framework for $\mathbf{AR}_4$ in further work, using a framework combining derivations for proofs with derivations for refutations, analogous to the approach in [35].

so simply in $\mathbf{AR}_4$: truth and falsity are made independent from each other, so that the speech-acts of assertion and rejection may have their own sets of preservation-conditions. Preservation of what, actually? If, as we argued for until here, the act of rejection does not depend from its dual opposite of assertion, some asymmetry should arise between the two logics of asserting and rejection. For instance, it is not because p is rejected that its negation $\neg p$ should be rejected at once. On the basis of our partition between assertion rules (whether positive about truth, or negative about falsity) and rejection rules, at least four different theories of preservation or not-preservation can be devised to characterize the logics of assertion and rejection. Namely:

Logic 1: theory of truth-preservation
Logic 2: theory of falsity-preservation
Logic 3: theory of truth-non-preservation
Logic 4: theory of falsity-non-preservation.[30]

The first two logics deal with assertion and strong rejection, respectively, whereas the last two ones with assertion and weak rejection.

4.3. Bilateralism and bivalentism

Smith and Incurvati [11] reconstruct Textor's arguments by means of the signed calculus, in which the symbols "+" and "–" appended to sentences express their assertion and rejection, respectively. Then, against Textor's exclusive interpretation of "no" as a statement of negative assertion in (P2), i.e. $+(\neg p)$, it is noted that:

> there is another possible explanation of the validity of [the deciduous tree argument] – one that does not require "no" to have the same meaning in (P1) and (P2). For here is another regimentation:
> (P1′) $+(\neg p \to q)$
> (P2′) $-p$
> (C′) Therefore: $+q$
> And this is an inference that the bilateralist recognizes as valid.[31]

However, two problems arise from the above explanation. First, what is the intended difference between $-p$ and $+(\neg p)$, if both lead to the same conclusion? Second, why should q be asserted once p is denied in (P2′)? We agree with the above distinction between two sorts of rejection $-(p)$ and $+(\neg p)$, against Textor's anti-rejectivist reading of "no". However, we disagree with the above argument. For, on our view, denying the antecedent entails that the entire conditional relation is cancelled. To be more precise about this game-type account of conditionality, let us see a conditional as a commitment, which comes to commit the speaker to assert the antecedent. Here is our alternative analysis of the preceding argument (P1′)-(C′) in terms of weak denial $-(p)$ in **QAS**:

(P1″) $\mathbf{a}_1(\neg p \to q) = 1 \Leftrightarrow (\mathbf{a}_1(\neg p) = 1 \Rightarrow \mathbf{a}_1(q) = 1)$
(P2″) $\mathbf{a}_1(p) = 0$

[30] Related constructions can be found in [8, 14, 27, 28, 29].
[31] In **QAS**, this is: (P1') $\mathbf{a}_1(\neg p \to q) = 1$; (P2') $\mathbf{a}_1(p) = 0$; (C') $\mathbf{a}_1(q) = 1$.

(C″) $\mathbf{a}_1(\neg p \rightarrow q) = 0$

There is no contradiction between (P1″) and (C″), despite the contrary appearance. This is so, because (P1″) is a definition of assertion rather than a commitment to its antecedent. A conditional is asserted only in cases when both antecedent and consequent are asserted, so, if the antecedent is not asserted, then the entire conditional is also not asserted, *without* invalidating the conditional. Given that the latter is denied by the second judgment (P2″), it follows from this inferential explanation of conditional speech-acts that the entire conditional is ruled out accordingly.[32]

Later, Incurvati & Smith [11] go on to contest Textor's exclusive use of “no” as a negative assertion while assuming a bivalentist question-answer game:

> In fact, the bilateralist can explain its validity in more basic terms. For, first, bilateral systems include the following rule:
> $(+\neg I)\ -\alpha;\ +(\neg\alpha)$
> And in passing, let's remark that bilateral systems will also include the converse rule.[33]

How can $(+\neg I)$ be accepted without entailing anti-rejectivism? It seems perfectly acceptable for a Bivalentist as:

$(-I)\ +(\neg\alpha);\ -\alpha$[34]

But, we hardly see how this should be the case for a non-Bivalentist. For example, $(+\neg I)$ may be accepted by Paraconsistentists, but not its converse; on the other hand, $(-I)$ is accepted by Paracompletists, but not its converse. So, either rejectivists are considered automatically to be bivalentist, or they are not, in which case the equation of rejectivism with bivalentism in Textor and in Incurvati and Smith should itself be rejected.[35]

4.4. Reduction and rejectivism

Somehow ironically, Frege's philosophy of language constitutes both support and hindrance to the rejectivist. As noted in Incurvati & Smith [11]:

> the bilateralist takes the unit of inference to be judgments. From this point of view, she is closer to Frege than to modern logicians.

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[32]The conditional is itself defined in terms of a biconditional “iff”. Does this entail a regression argument, insofar as conditional is defined by its own defining words in the metalanguage? To deal with this problem, which is similar to that with Achille and the Tortoise (about the process of detachment in Modus Ponens), we have to insist upon the notion of act in the speech act of assertion. In this respect, the “$\Rightarrow$” is a metalinguistic symbol of ensuing action after a preceding speech act.

[33]In $\mathbf{AR}_4$, this is: $(+\neg I)$ $\mathbf{a}_1(p) = 0$; $\mathbf{a}_1(\neg p) = \mathbf{a}_2(p) = 1$.

[34]In **QAS**, this is: $\mathbf{a}_1(\neg\alpha) = \mathbf{a}_2(\alpha) = 1$; $\mathbf{a}_1(\alpha) = 0$.

[35]Whilst it would take us beyond the discussion of this paper, we should also note that our approach to bilateralism is well positioned to deal with revenge paradoxes that are identified in relation to self-reference in more standard bilateral logics [15], since we might invoke our dynamic approach to speech acts to deal with these. We leave this work for a further paper, however, and thank an anonymous reviewer for drawing our attention to this.

This is in accord with our view that each sentence is a proto-statement whose meaning depends upon its use in an inferential process, and also with our view that "$\neg p$" is just a proto-statement expressing the falsity questioned of p. In **QAS**, these form the content of the second question $\mathbf{q}_2(p)$ characterizing the meaning of p in $\mathbf{Q}(p) = \langle \mathbf{q}_1(p), \mathbf{q}_2(p) \rangle$. So, a speaker asserts p negatively by saying "yes" to whether p is not the case; they say "no", otherwise. Moreover, let us recall that our valuation is a non-Fregean one in assuming the independence of truth and falsity as two possible forms of assertion. This second feature provides a way of avoiding Frege's dismissal of rejectivism. The latter is considered by Incurvarti & Smith [11], as a challenge to the rejectivist:

> As a result, however, she has to deal with the difficulties Frege famously faced in explaining suppositional reasoning: if judgments are the units of inference, what do we do when we assume something for reductio?

In our framework, this provides justification for an alternative definition of a strengthened conditional in $\mathbf{AR}_4$, as an operation prefiguring the proto-statement of inference. For this purpose, we propose an explanation of conditionality on the basis of a comparative analysis of Modus Ponens and Modus Tollens (or "reductio") as follows. There can be at least three different versions of Modus Ponens, together with their interpretation in terms of questions-answers in $\mathbf{AR}_4$:

MP1. $\vdash ((p \to q) \wedge p) \to q$

$\mathbf{AR}_4$. $\mathbf{a}_1((p \to q) \wedge p) \to q)) = 1$

MP2. $(p \to q), p \vdash q$

$\mathbf{AR}_4$:. $(\mathbf{a}_1(p \to q) = \mathbf{a}_1(p) = 1) \Rightarrow \mathbf{a}_1(q) = 1$

MP3. $\vdash p \to q, \vdash p, \vdash q$

$\mathbf{AR}_4$:. $(\mathbf{a}_1(p \to q) = 1, \mathbf{a}_1(p) = 1) \Rightarrow \mathbf{a}_1(q) = 1$

It is the case that MP1 is invalid in $\mathbf{AR}_4$, whenever $\mathbf{a}_1(p)$ or $\mathbf{a}_1(q) = 0$. As for MP2 and MP3, they are both valid in $\mathbf{AR}_4$ and equivalent with each other. The main advantage of the latter is that these do not include further logical constants in addition with conditional, thereby providing a "pure" definition in terms of committal speech-acts of assertion. At the same time, our formalism shows that the meaning of Modus Tollens is much more complex and may give rise to more interpretations in $\mathbf{AR}_4$:

MT1. $\vdash ((p \to q) \wedge \neg q) \to \neg p$

$\mathbf{AR}_4$:. $\mathbf{a}_1((p \to q) \wedge \neg q) \to \neg p) = 1$

MT2. $(p \to q), \neg q \vdash \neg p$

$\mathbf{AR}_4$:. $(\mathbf{a}_1(p \to q) = \mathbf{a}_1(\neg q) = 1 \Rightarrow \mathbf{a}_1(\neg p) = 1$

MT3. $\vdash p \to q, \vdash \neg q, \vdash \neg p$

$\mathbf{AR}_4$:. $(\mathbf{a}_1(p \to q) = 1, \mathbf{a}_1(\neg q) = 1) \Rightarrow \mathbf{a}_1(\neg p)$

MT4. $\vdash p \to q, \dashv q, \dashv p$

$\mathbf{AR}_4$:. $\mathbf{a}_1(p \to q) = 1, \mathbf{a}_1(q) = 0 \Rightarrow \mathbf{a}_1(p) = 0$

MT5. $\dashv p \to q, \dashv p, \dashv q$

AR$_4$:. $\mathbf{a}_1(p \to q) = \mathbf{a}_1(p) = 0 \Rightarrow \mathbf{a}_1(q) = 0$

MT6. $\dashv p \to q, \dashv q, \dashv p$

AR$_4$:. $\mathbf{a}_1(p \to q) = \mathbf{a}_1(q) = 0 \Rightarrow \mathbf{a}_1(p) = 0$

We think that the definition of Modus Tollens that coheres best with the account of the conditional given above, whilst also doing justice to rejectivism, is MT4. Take, for example, MT5 and MT6. Whilst these engage with the common actions of denial upon the components p, q of the conditional $p \to q$, and negation is not required in these interpretations of Modus Tollens, MT5 does not seem to render the meaning of Modus Tollens adequately. The second term of the premise is not the initial consequent q, but the antecedent p. So, a confusion seems to emerge between Modus Tollens and Modus Ponens, since the conditions under which the two Modi hold rely upon one and the same component in MT5 and MT6 – the antecedent p, which is asserted in MP and denied in MT. MT4, on the other hand, requires that, if I assert that “if p, then q”, and if I reject q, then I must reject p, also. Whilst this interpretation requires us to include both assertion and rejection within the same inferential process, this dynamic corresponds to that identified with the failing bet analysis of conditionals given above. If developed in terms of a dialogue between two interlocuters, then this “mixed” interpretation may become more intuitive as a way of thinking of the assertion of a conditional on behalf of proponent, which leads to a later rejection of the antecedent once the opponent has provided a successful attack of the consequent. We leave this issue open, for further development.

5. Conclusion

We have constructed a generalised account of rejectivism by developing the formal framework of **QAS** and its ensuing four-valued logic of acceptance and rejection, **AR**$_4$. By considering and outlining the flaws in Textor’s arguments against rejectivism, we have provided a novel approach that challenges long-standing views regarding the non-embeddability and non-iteratibility of force indicators. The formalism that we offer not only provides a solid foundation for a dialogical approach to logic, but we also highlighted several advantages of our approach to the conditional and an ability to account for distinct pragmatic models of “correctness”.

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